

The exam has 100 pts. Closed book, no calculators. Write all answers and show all work inside your blue book. Read carefully.

**Problem 1 (5 pts).**

Convert the following to the specified number system.

- (a)  $FAB_{16}$  to Octal.
- (b)  $1010\ 1101_2$  to Decimal.
- (c)  $808_{10}$  to Binary.
- (d)  $65_{10}$  to Hexadecimal.
- (e)  $210_3$  to Decimal.

**Problem 2 (5 pts).**

Perform the following computation,  $18_{10} - 32_{10}$ , in a two's complement binary number system. Verify that your answer is correct by converting your answer back to decimal.

**Problem 3 (14 pts).**

Consider the following boolean expression,

$$X = \overline{\overline{A + B}} + A(\overline{\overline{B + BC}}) + (A + \overline{C})\overline{A}BC\overline{D}$$

- (a) Write  $X$  in sum-of-products form.
- (b) Convert  $X$  to a truth-table.
- (c) Use a Karnaugh-map to simplify  $X$  from its truth table.
- (d) Draw a logic diagram for the simplified expression. Label all inputs and outputs.

**Problem 4 (8 pts).**

- (a) Write the truth-table and corresponding boolean expression for a 2:1 multiplexer.
- (b) Draw the logic diagram of a 4:1 mux using 2:1 muxes.
- (c) Draw the logic diagram of a 16:1 mux using 4:1 muxes.

**Problem 5 (20 pts).**

In this problem, you will build a digital comparator that takes two  $N$ -bit binary numbers,  $A$  and  $B$ , and sets separate pins  $E$ ,  $G$ , or  $L$  to high if  $A = B$ ,  $A > B$ , or  $A < B$ , respectively.

- (a) Write the truth-table for a *half-comparator* which takes as input two 1-bit binary numbers,  $A$  and  $B$ , and outputs  $E$ ,  $G$ , and  $L$ .
- (b) Determine boolean expressions of  $E$  and  $G$ . Express  $L$  in terms of  $E$  and  $G$ .

- (c) A 1-bit *full-comparator* optionally takes as input the  $E$  and  $G$  results from another comparator (acting on more significant bits). Determine boolean expressions for the  $E$ ,  $G$ , and  $L$  outputs of a 1-bit full-comparator in terms of  $A$ ,  $B$ ,  $E_{in}$ , and  $G_{in}$ .
- (d) Draw a logic diagram for the 1-bit full-comparator. Draw a corresponding symbol to represent it.
- (e) Draw a logic diagram for a 4-bit full-comparator, comparing  $A_3A_2A_1A_0$  to  $B_3B_2B_1B_0$  (left MSB), using 1-bit full comparators. Label all inputs and outputs. Do not leave input pins floating.

**Problem 6 (4 pts).**

Describe similarities and differences between the *Gated D-latch* and the *Master-Minion D Flip-Flop*.

**Problem 7 (24 pts).**

In this problem, you will build a Mod-4 binary up/down counter with positive edge-triggered D flip-flops. The direction of counting is to be controlled by a selection input  $S$ , where  $S = 0$  denotes up counting and  $S = 1$  denotes down counting. Counting must loop around if inputs are not changed.

- (a) Construct a state transition diagram for your counter.
- (b) Convert your state transition diagram to a table, labeling current states, inputs, control variables, and next states. Determine values for your control variables to achieve the desired transitions.
- (c) Determine simplified boolean expressions for your control variables.
- (d) Draw a logic diagram of your circuit. Label the clock,  $S$ , and your state variables clearly.

**Problem 8 (20 pts).**

Consider the circuit in Figure 1, consisting of a Mod-4 binary up/down counter and a JK flip-flop, where the counter operates as specified in the previous problem. Suppose that the circuit is initialized with the counter in the  $Q_1Q_0 = 2_{10}$  state and the JK FF in the 0 state.

- (a) Is this circuit synchronous or asynchronous? Why or why not?
- (b) Draw a timing diagram of  $Q_1$ ,  $Q_0$ ,  $Y$ , and  $S$  with 8 positive edges from the CLK, starting from the above mentioned initialization.
- (c) Describe the behavior of the circuit.

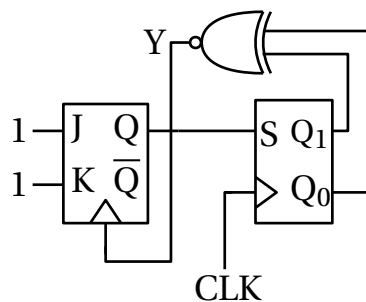


Figure 1: Binary up/down counter with external JK flip-flop.